Exercise 51

Find an equation of the tangent line to the curve at the given point.

$$y = 2^x, \quad (0, 1)$$

Solution

A point on the tangent line is known, so all that's needed is its slope. Take a derivative of the given function

$$y' = \frac{d}{dx}(2^x) = \frac{d}{dx}(e^{\ln 2^x}) = \frac{d}{dx}(e^{x \ln 2}) = e^{x \ln 2} \cdot \frac{d}{dx}(x \ln 2) = e^{x \ln 2} \cdot (\ln 2)$$

and evaluate it at $x = 0$.

$$y'(0) = e^0 \ln 2 = \ln 2$$

Therefore, the equation of the tangent line to $y = 2^x$ at $(0, 1)$ is

$$y - 1 = (\ln 2)(x - 0).$$

Below is a graph showing the function and the tangent line.